The Fundamental Theorem of Algebra

An *n*th degree polynomial function has exactly *n* zeros. *

$f(x) = 3x^4 - 2x^3 - 4x + 9$ has 4 zeros

$f(x) = 2x^6 + 5x^5 - 4x^4 + 9x^2 + x - 7$ has 6 zeros

$f(x) = -2x^2 - 4x + 9$ has 2 zeros

$f(x) = 3x^2 - 2x^3 - 4x^5 + 9x - 3$ has 5 zeros

* Each repeated solution must be counted as a separate solution.

$f(x) = x^2 + 4x + 4$
$= (x+2)(x+2)$
$x = -2, -2$

Find all the zeros of $f(x) = x^5 - 4x^3 + x^2 - 4$

1. List possible zeros: $\dfrac{\pm 1, \pm 2, \pm 4}{\pm 1} = \pm 1, \pm 2, \pm 4$

2. Test possible zeros (look at graph first if you want to.)

$$\begin{array}{r|rrrrrr} 1 & 1 & 0 & -4 & 1 & 0 & -4 \\ & \downarrow & 1 & 1 & -3 & -2 & -2 \\ \hline & 1 & 1 & -3 & -2 & -2 & -6 \end{array}$$

$$\begin{array}{r|rrrrrr} -1 & 1 & 0 & -4 & 1 & 0 & -4 \\ & & -1 & 1 & 3 & -4 & 4 \\ \hline & 1 & -1 & -3 & 4 & -4 & 0 \end{array}$$

$(x+1)$ is a factor

Try -1 again

$$\begin{array}{r|rrrrr} -1 & 1 & -1 & -3 & 4 & -4 \\ & & -1 & 2 & 1 & -5 \\ \hline & 1 & -2 & -1 & 5 & -9 \end{array}$$

-1 is not a repeated solution

$$\begin{array}{r|rrrrr} 2 & 1 & -1 & -3 & 4 & -4 \\ & & 2 & 2 & -2 & 4 \\ \hline & 1 & 1 & -1 & 2 & \cancel{0} \end{array}$$

$(x-2)$ is a factor.

Try 2 again

$$\begin{array}{r|rrrr} 2 & 1 & 1 & -1 & 2 \\ & & 2 & 6 & 10 \\ \hline & 1 & 3 & 5 & 12 \end{array}$$

2 is not a repeated solution.

$$\begin{array}{r|rrrr} -2 & 1 & 1 & -1 & 2 \\ & & -2 & 2 & -2 \\ \hline & 1 & -1 & 1 & 0 \end{array}$$

-2 is a zero

$(x+2)$ is a factor

$$f(x) = \underbrace{(x+1)}_{\substack{x+1=0 \\ x=-1}}\underbrace{(x-2)}_{\substack{x-2=0 \\ x=2}}\underbrace{(x+2)}_{\substack{x+2=0 \\ x=-2}}(x^2 - x + 1)$$

$x^2 - x + 1$

Use Quad. Form

$$x = \frac{1 \pm \sqrt{1 - 4(1)(1)}}{2(1)}$$

$$= \frac{1 \pm \sqrt{-3}}{2} = \frac{1 \pm i\sqrt{3}}{2}$$

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10.) $f(x) = x^4 - 6x^3 + 7x^2 + 6x - 8$

(x-1) is a factor

$$
\begin{array}{r|rrrrr}
1 & 1 & -6 & 7 & 6 & -8 \\
 & \downarrow & 1 & -5 & 2 & 8 \\
\hline
 & 1 & -5 & 2 & 8 & 0
\end{array}
$$

$$
\begin{array}{r|rrrr}
1 & 1 & -5 & 2 & 8 \\
 & & 1 & -4 & -2 \\
\hline
 & 1 & -4 & -2 & 6
\end{array}
$$

(x+1) is a factor

$$
\begin{array}{r|rrrr}
-1 & 1 & -5 & 2 & 8 \\
 & & -1 & 6 & -8 \\
\hline
 & 1 & -6 & 8 & 0
\end{array}
$$

$f(x) = (x-1)(x+1)(x^2 - 6x + 8)$

$= (x-1)(x+1)(x-4)(x-2)$

$x-1=0$ $\boxed{x=1}$ $\quad$ $x+1=0$ $\boxed{x=-1}$ $\quad$ $x-4=0$ $\boxed{X=4}$ $\quad$ $x-2=0$ $\boxed{x=2}$

18.) $x^4 - 8x^3 + 14x^2 + 8x - 15$

$$\begin{array}{r|rrrrr} 5 & 1 & -8 & 14 & 8 & -15 \\ & & 5 & -15 & -5 & 15 \\ \hline & 1 & -3 & -1 & 3 & 0 \end{array}$$

$(x-5)$ is a factor

$$(x-5)(x^3 - 3x^2 - x + 3)$$

$$\begin{array}{r|rrrr} 1 & 1 & -3 & -1 & 3 \\ & \downarrow & 1 & -2 & -3 \\ \hline & 1 & -2 & -3 & 0 \end{array}$$

$(x-1)$ is a factor

$$f(x) = (x-5)(x-1)(x^2 - 2x - 3)$$
$$f(x) = (x-5)(x-1)(x-3)(x+1)$$

$x = 5$ $x = 1$ $x = 3$ $x = -1$

23.) $5x^4 + 6y^3 - 84x^2 + 58x + 15$

$$\begin{array}{r|rrrrr} 3 & 5 & 6 & -84 & 58 & 15 \\ & \downarrow & 15 & 63 & -63 & -15 \\ \hline 1 & 5 & 21 & -21 & -5 & \cancel{0} \\ & \downarrow & 5 & 26 & 5 & \\ \hline & 5 & 26 & 5 & 0 & \end{array}$$

$(x-3)$ is a factor

$(x-1)$ is a factor

$$f(x) = (x-3)(x-1)(5x^2 + 26x + 5)$$
$$= (x-3)(x-1)(5x+1)(x+5)$$

$x - 3 = 0$ $x - 1 = 0$ $5x + 1 = 0$ $x + 5 = 0$

$x = 3$ $x = 1$ $5x = -1$ $x = -5$

$x = -\frac{1}{5}$